SPECIFIC FEATURES OF PARAMETRIC EXCITATION OF THERMOMECHANICAL VIBRATION OF A WIRE HEATER

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An experimental study and a theoretical analysis are performed for excitation of vibrations of a thin heater in order to enhance heat transfer. It is established that to obtain a practically significant rate of the parametric enhancement of such vibrations, the fulfillment of additional conditions, determined by the character of temperature modulations, is required.

It is well known that the rate of heat transfer from a thin heater increases considerably if it is set in oscillatory motion [1, 2]. To excite high-rate transverse vibrations of a hot tense wire (a string), in [3] we recommended producing temperature oscillations to induce periodic variations (modulations) in the string tension σ. Should the ratio of the frequency ω of the modulations to the natural frequency ω_0 of the string be approximately equal to 2/n, where n = 1, 2, ..., then, according to parametric resonance theory, thermomechanical oscillations (TMO) of the wire heater must be excited, which are associated with an increase in the heat transfer coefficient α. Our previous experiments [4] on the whole validated this idea, but in many cases the rate of the TMO build-up appears to be so low that the expediency of applying this effect in engineering becomes doubtful. Therefore, we have faced the task of clarifying quantitative regularities of the parameteric TMO and of identifying the conditions for the onset of *real* parametric resonance (RPR), with which the TMO swing builds up at a practically sufficient rate.

To solve the above-stated problem, we carried out numerous experiments in which the wire heaters were fed by a Π-shaped pulse current of meander type, whose pulse frequency could be varied smoothly. Skipping the description of experimental rigs and measurement techniques (we hope to do this in a separate paper), we would only like to note that, with the aid of a double-beam oscilloscope, we had the possibility of tracking both the character of temperature oscillations $\Theta(t)$ in the string and its transverse oscillations $U(x,t)$.

The analysis of the oscillograms allowed the formulation of the following basic experimental results.

1. The temperature oscillations $\Theta(t)$ of the string in pulse heating occur at a frequency ω of the current pulses by a peculiar relaxation law, i.e., for each period $T = 2\pi/\omega$ there are two branches of the curve $\Theta(t)$ equal in length, viz., ascending (I) and descending (II), which conform to the pulse and break of the current $I(t)$ (Fig. 1).

2. The real parametric excitation of the wire heater TMO occurs only at even resonance numbers, i.e., when n = 2, 4, ..., 2k. Thus, the frequency condition of the RPR onset takes the form

$$\omega \simeq \frac{1}{k}\omega_0 \, (k = 1, 2, \ldots). \tag{1}$$

3. Throughout the parametric enhancement of the TMO, the character of temperature oscillations $\Theta(t)$ and the initial phase displacement $\Delta\varphi$ vary continually until a mode of parametric self-oscillations is established.

4. The established TMO are characterized by the phase displacement $\Delta\varphi \simeq \pi/2$, identical for all n, and by a peculiar type of temperature oscillations $\Theta^*(t)$, noticeably differing from the initial $\Theta_0(t)$. To clear up physical causes of these unusual results, we first of all take into account that oscillatory phenomena differing in nature interact in the system considered, viz., the mechanical oscillations of the string $U(x,t)$ with the natural frequency ω_0, which occur at the modulation frequency ω of the feeding current pulses $I(t)$, and the temperature oscillations of the string $\Theta(t)$ and the periodic variations in its tension $\sigma(t) = \sigma_0 - c\Theta(t)$ generated by them (c is constant).

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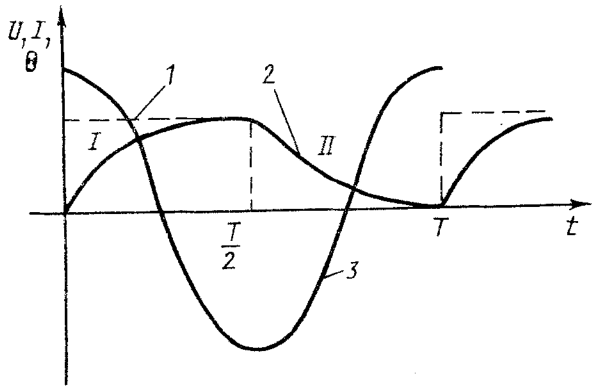

Fig. 1. Temperature oscillations in the string heated by pulse current: 1) current pulses I(t); 2) plot of the string temperature as a function of the TMO time Θ(t); 3) transverse oscillations of the string U(t).

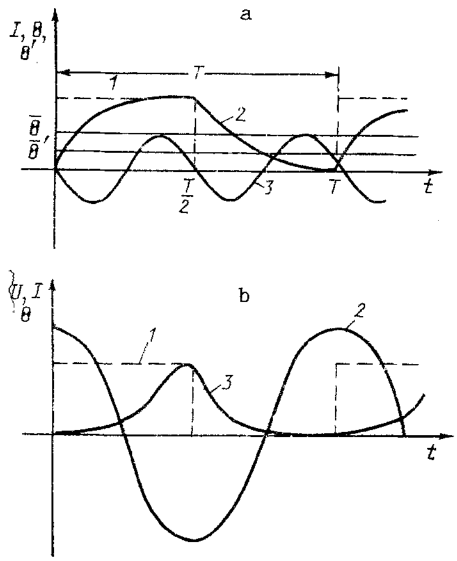

Fig. 2. Thermomechanical and mechanothermal oscillations of a vibrating string heated by pulse current: a) form of the curves [1) I(t); 2) Θ(t); 3) Θ'(t)] at equal frequencies $\omega = \omega_0$; b) mechanical vibrations of a string U(t) (2) and resulting temperature modulations $\Theta^*(t) = \Theta + \Theta'$ (3).

It is understandable that the key role in the process of parametric enhancement of the TMO is played by the character of the temperature oscillations Θ(t), whose branches I and II are close to exponential, so that their steepest rise or fall takes place in the beginning of the corresponding semiperiod T/2 (Fig. 1).

An important role is also played by the initial phase displacement $\Delta\varphi$ between Θ(t) and the transverse oscillations U(t) of the string (U(t) is the deflection averaged over its length). We may define the phase displacement using the following hypothesis (which is verified by experiments with the TMO, excited by temperature oscillations of various types): as a result of the RPR onset in the system the phase displacement $\Delta\varphi$ is automatically established, to which a maximal rate of the parametric increase of the oscillatory energy corresponds for this type of modulation.

Using the above hypothesis and the familiar relationship for the rate of parametric increment of the energy E [2]

$$\frac{dE}{dt} = \frac{U^2}{2}\,\frac{d\sigma}{dt}, \qquad (2)$$

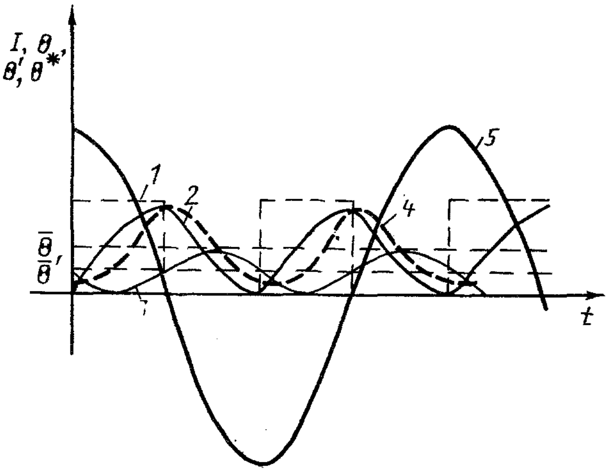

Fig. 3. Forms of the curves $\Theta(t)$, $\Theta'(t)$, and $\Theta^*(t)$ for $n = 1$: 1) current pulses $I(t)$; 2) thermomechanical oscillations $\Theta(t)$; 3) mechanothermal oscillations $\Theta'(t)$; 4) resulting temperature modulations $\Theta^*(t)$; 5) transverse oscillations of a string $U(t)$.

we find that, near $t = T/2$, where $d\sigma/dt = -d\Theta/dt$ attains a maximum, the string deflection U must be equal to the peak value. Hence, as is clear from Fig. 1, the oscillations $\Theta(t)$ lag behind $U(t)$ by the angle $\varphi \approx \pi/2$ in phase (precisely such initial phase displacement is observed in experimental oscillograms).

Now let us estimate the energy supply over the modulation period E_T. For the first semiperiod ($0 \leq t \leq T/2$), the temperature Θ rises exponentially (Fig. 1), whereas the string tension is reduced ($d\sigma/dt < 0$); therefore, $E_I < 0$, i.e., energy outflow occurs. In the second semiperiod, $d\sigma/dt > 0$ and, consequently, $E_{II} > 0$. The total parametric supply of energy for the period is $E_T = E_I + E_{II}$. However, since the rise and the fall of the temperature $\Theta(t)$ obey nearly the same exponential laws, then, apparently, $-E_I \approx E_{II}$, so that $E_T \approx 0$. This implies that no real parametric enhancement of the TMO must occur with the considered type of thermal oscillations $\Theta(t)$. Experiments, however, show that if the frequency condition is fulfilled

$$\omega \simeq \frac{2}{n}\omega_0 \tag{3}$$

and the number n is even ($n = 2, 4, \ldots$), the RPR emerges in the system.

It remains to be suggested that there is one more oscillatory process in the system, which decreases the energy outflow in the first semiperiod E_I and increases its supply in the second semiperiod E_{II}.

The analysis has conviced us that this function is performed by so-called mechanothermal oscillations (MTO), studied in detail in [5]. The MTO phenomenon is underlayed by the well ascertained fact that the heat transfer coefficient α in a hot wire moving in an air or liquid medium increases by the quantity $\Delta\alpha = k'\sqrt{v}$, where k' is a constant dependent on the wire and medium properties, and v is the movement velocity.

In low-frequency harmonic oscillations of a thin wire, the coefficient α undergoes, in a first approximation, pulsations at the frequency $\omega' = 2\omega_0$ [2]

$$\alpha(t) = \alpha_0 + \Delta\alpha_0(1 + \cos 2\omega t). \tag{4}$$

As a result, the time-averaged temperature of the string, which so far has been equal to $\bar{\Theta}$, decreases by a certain quantity $\Delta\Theta_0$, dependent on the amplitude and the frequency ω_0 of transverse oscillations of the wire heater. Relative to this average value $\bar{\Theta}' = \bar{\Theta} - \Delta\Theta_0$, thermomechanical oscillations occur

$$\Theta'(t) = \bar{\Theta}' - \Delta\Theta_0 \sin(2\omega_0 t - \varphi). \tag{5}$$

As study [5] revealed, because of thermal inertia of the string, the pulsations $\Theta'(t)$ are out of phase with respect to U(t) by the angle $\Delta\varphi \simeq \pi/2$, so that the initial phase displacement between $\Theta'(t)$ and $\Theta(t)$ is equal to π.

Clearly, the MTO $\Theta'(t)$ change the character of the thermal oscillations $\Theta(t)$ and, hence, of the tension modulations $\sigma(t)$. It remains for us to determine how the relationship between the frequencies $\omega' = 2\omega_0$ of the MTO $\Theta'(t)$ and ω of the TMO $\Theta(t)$ affects the parametric energy increase over the period E_T. For this end, we first assume that $\omega = \omega_0$, i.e., that the number n in Eq. (3) is equal to 2. In this case, for each meander period there are two corresponding waves of $\Theta'(t)$ (Fig. 2a); therefore, a superposition of $\Theta(t)$ and $\Theta'(t)$ results in the curve $\Theta^*(t) = \Theta + \Theta'$ (Fig. 2b).

Evidently, in the beginning of the 1st and in the end of the 2nd quarter of the period T, where $|U(t)|$ is large, the curve $\Theta^*(t)$ is very gently sloping ($d\Theta^*/dt = -d\sigma/dt \approx 0$), whereas, near $t = T/4$, $d\Theta^*/dt$ is large and positive, and, correspondingly, $d\sigma/dt < 0$, but here $U(t) \simeq 0$. Therefore, the energy outflow over the 1st semiperiod E_I appears to be insignificant. In the 2nd semiperiod, the curve $\Theta^*(t)$ in the beginning of the 3rd quarter descends more steeply than $\Theta(t)$, and, since $U \cong U_{max}$ near $t = T/2$, the energy inflow E_{II} is increased. Therefore, the parametric increment of the energy over the entire period proves to be appreciably above zero, i.e., the RPR emerges in the system. To consider the cases with $n \neq 2$, it is sufficient to take into account that, in conformity with Eq. (5), when thermal pulsations $\Theta'(t)$ are superposed on $\Theta(t)$, the first half-wave should be subtracted from, and the second added to, $\Theta(t)$. Then it is easy to verify that the energy addition for odd (in contradistinction to even) n is insignificant. As an example, Fig. 3 plots $\Theta(t)$, $\Theta'(t)$, and $\Theta^*(t)$ for $n = 1$.

CONCLUSIONS

1. When a wire heater is fed by a pulse current, complex temperature modulations, continually varying in time, arise from the superposition of thermomechanical and mechanothermal oscillations of a tense wire.

2. The fulfillment of the frequency condition, needed for the onset of the parametric resonance, appears to be insufficient for obtaining a perceptible rate of the vibration enhancement. To this end, certain additional requirements dependent on the modulation character should be met.

3. If a tense wire is heated by a meander-type pulse current, the real parametric build-up of thermomechanical oscillations of a wire heater, accompanied by a noticeable increase in the heat transfer coefficient, occurs only on even orders n of the resonance.

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